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SHORTER COMMUNICATIONS

SOME NEW RESULTS FOR CONJUGATED HEAT TRANSFER IN A FLAT PLATE

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NOMENCLATURE

C, constant depending on the flow type and boundary condition in equations (1)–(3);
C'', heat capacity/surface of half plate;
h_c, heat-transfer coefficients for convection;
k, thermal conductivity of the fluid;
k_p, thermal conductivity of the plate;
$2l$, thickness of the plate;
L, length of the plate;
m, n, exponents depending on the flow type;
Pr, Prandtl number;
q, heat flux density;
Re, Reynolds number $U_\infty x/\nu$, $Re_L = U_\infty L/\nu$;
s, dummy variable of the plate length $0 \leqslant s \leqslant x$;
T, surface or plate temperature;
T_∞, fluid temperature;
T_s, steady-state plate temperature;
ΔT_j, temperature change during Δt;
t, time;
Δt, time interval;
U_∞, freestream velocity;
x, coordinate along the plate;
Δx, subinterval length.

Greek symbols

β, γ, exponents depending on the flow type and boundary condition;
ν, kinematic viscosity;
ϕ'', internal energy generation/surface.

1. INTRODUCTION

THE PROBLEMS of the mutual thermal effect of the solid body and the fluid flow, i.e. conjugated heat-transfer problems, have received some attention during recent years. Because the temperature distributions of the fluid and the solid are coupled together the arising problems are very complex. For a flat plate in forced flow there exist analytical [1–3], numerical [4–6] and experimental works [7] and often some approximations have been done, for instance a slug flow assumption.

This paper deals with conjugated heat transfer in a flat plate. An approximate method is presented for calculating heat transfer from a flat plate in forced flow. The results are compared with experimental data and previous results obtained in [4] for the case of combined convective heat exchange with the environment, conduction in the plate and internal heat sources.

2. CONVECTIVE HEAT TRANSFER FOR PRESCRIBED SURFACE TEMPERATURE AND HEAT-FLUX DISTRIBUTIONS

If the surface temperature of a plate in Fig. 1 is known (solid line), the greatest interest lies in the heat flux density corresponding to that temperature. For arbitrary surface

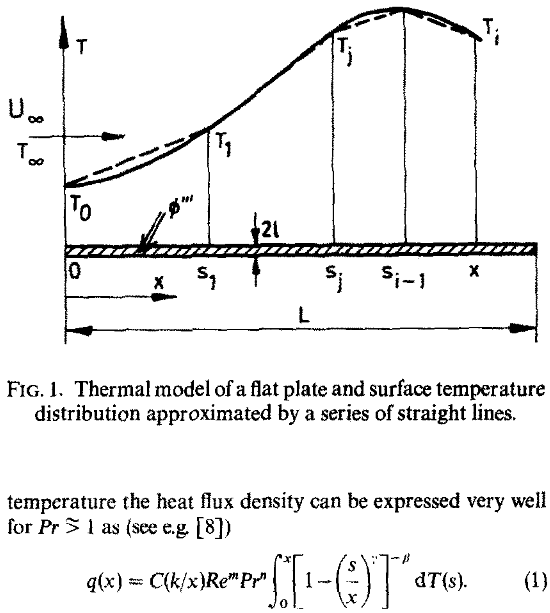

FIG. 1. Thermal model of a flat plate and surface temperature distribution approximated by a series of straight lines.

temperature the heat flux density can be expressed very well for $Pr \gtrsim 1$ as (see e.g. [8])

$$q(x) = C(k/x)Re^m Pr^n \int_0^x \left[1-\left(\frac{s}{x}\right)^\gamma\right]^{-\beta} \mathrm{d}T(s). \qquad (1)$$

The constants C, m, n, γ and β are given for laminar and turbulent flows in Fig. 2. Another problem is presented when the heat flux distribution is given. Then we are interested in the corresponding surface temperature, which can be found [4, 8] from

$$T(x)-T_\infty = \frac{Re^{-m}Pr^{-n}}{Ck}\int_0^x \left[1-\left(\frac{s}{x}\right)^\gamma\right]^{\beta-1} q(s)\,\mathrm{d}s. \qquad (2)$$

Equation (2) results from the solution of a Volterra integral equation composed of equation (1). Instead of equation (2) also a simpler expression can be obtained by applying directly the energy integral equation to the problem of the step in heat flux [6]

$$T(x)-T_\infty = \frac{Re^{-m}Pr^{-n}}{C}\left(\frac{x}{k}\right)\int_0^x \left(1-\frac{s}{x}\right)^\beta \mathrm{d}q(s). \qquad (3)$$

The differences between the results obtained using approximate expressions (2) and (3) are according to [4] very small.

3. HEAT TRANSFER FROM A FLAT PLATE IN THE CASE OF COMBINED CONDUCTION AND CONVECTION

Consider a thin plate of finite length in a fluid flow with uniform internal generation of thermal energy in Fig. 1. Externally heat is transferred to a fluid by convection and there can also be conduction along the plate. If conduction is taken to be one-dimensional then the temperature distri-

Flow type	Thermal condition	C	γ	β	m	n
laminar	temperature	0.332	3/4	1/3	1/2	1/3
	heat flux	0.460 1.605	1 3/4			
turbulent	temperature	0.0296	9/10	1/9	4/5	3/5
	heat flux	0.0307 0.301	1 9/10			

FIG. 2. Constants in equations (1)–(3) for various flow situations and boundary conditions.

bution of the plate is governed by integro-differential [4] resulting from equation (2)

$$T(x)-T_\infty = \frac{Re^{-m}Pr^{-n}}{Ck}\int_0^x \left[1-\left(\frac{s}{x}\right)^\gamma\right]^{\beta-1} \times \left[k_p l\frac{\mathrm{d}^2T}{\mathrm{d}s^2}+\phi''\right]\mathrm{d}s. \quad (4)$$

Instead of solving equation (4), the temperature of the plate can be found using a simple method developed by the author in [9]. As a first step let us assume that the temperature distribution of the plate is the one presented by a solid line in Fig. 1. The corresponding convective heat flux is obtained from equation (1) and integration is easy if the true temperature is approximated by a series of dotted straight lines in Fig. 1. After integration we get

$$q(x) = C(k/x)Re^m Pr^n\left\{T_0-T_\infty+k_1x \times G\left(\frac{s_1}{x}\right)+k_2x\left[G\left(\frac{s_2}{x}\right)-G\left(\frac{s_1}{x}\right)\right] + \ldots + k_ix\left[G(1)-G\left(\frac{s_{i-1}}{x}\right)\right]\right\}, \quad (5)$$

where $k_j = (T_j - T_{j-1})/(s_j - s_{j-1})$ and

$$G\left(\frac{s_j}{x}\right) = \int_0^{s_j/x}[1-\alpha^\gamma]^{-\beta}\,\mathrm{d}\alpha. \quad (6)$$

When both the temperature T_j and the heat flux q_j are known at the point s_j, it is possible to calculate the heat-transfer coefficient caused by convection

$$h_{cj} = q_j/(T_j - T_\infty). \quad (7)$$

The combined action of convection and conduction can be considered after calculating the heat-transfer coefficient. Assuming one-dimensional conduction and taking the length of every subinterval as Δx, the finite-difference equation for the temperature of the point s_j can be written as

$$T_{j-1}-\left[2+h_{cj}\frac{\Delta x^2}{k_pl}\right]T_j+T_{j+1} = -\frac{\Delta x^2}{k_pl}[\phi''+h_{cj}T_\infty]. \quad (8)$$

The temperature of the plate is thus governed by the system of linear equations expressed in matrix notation as

$$AT = b. \quad (9)$$

If the initial guess of the temperature is correct, the solution of equation (9) does not differ from it: if not, the solution is substituted in equation (5) and a new temperature profile is calculated and so on.

The problem of Fig. 1 can also be approached by employing the heat flux distribution as a starting point and using either equation (2) or (3). Because both expressions have a similar level of accuracy, equation (3) is used. If heat flux distribution is first guessed and then approximated by straight lines as is the temperature in Fig. 1, the expression for the local plate temperature is obtained from equation (3), which gives

$$T(x)-T_\infty = \frac{Re^{-m}Pr^{-n}x}{C}\frac{1}{k}\left\{q_0+\frac{x}{1+\beta}\sum_{j=1}^{i} \times k_j\left[\left(1-\frac{s_{j-1}}{x}\right)^{1+\beta}-\left(1-\frac{s_j}{x}\right)^{1+\beta}\right]\right\}, \quad (10)$$

where $k_j = (q_j - q_{j-1})/(s_j - s_{j-1})$. Using the known temperatures and heat fluxes, the heat-transfer coefficient can be calculated and coupled temperature distribution is obtained again by solving the system of equations (9). After calculating the temperature the corrected heat flux is obtained from

$$q_j = h_{cj}(T_j - T_\infty). \quad (11)$$

Substituting the heat fluxes of equation (11) in equation (10), the new temperature distribution is obtained and so on.

In [9] also heat exchange to a large constant temperature environment by radiation is included. It has been found that the results are the same using both the temperature and heat flux as a starting point. Also it has been observed that results agree with [4] when heat is transferred only by convection and radiation. But as to conduction there is a great difference between the results in [4] and [9], where also experimental temperature distributions along the plate have been presented.

Figure 3 presents calculated and measured plate temperature distributions for a copper plate with a length of 0.3 m and thickness 0.3 mm in absence of radiation. The plate was placed in the free exhaust jet of a wind tunnel. A uniformly distributed heat generation was achieved by means of an electric current. The temperature distribution of the plate was obtained by using an infrared radiation camera. In the infrared radiation unit used, linescanning was possible and the radiation of one line could be displayed on an oscilloscope. Also transient cases could be easily measured. In order to measure the radiation, the plate surface, e.g. a reflecting aluminium surface, must be painted. If the purpose is to investigate the combined effect of only convection and conduction, only thin lines are painted in a transverse direction to the flow. The probable error in the surface temperature is estimated to be ± 0.4 K. Further details can be found in [9].

The results in Fig. 3 are very surprising near the leading edge, because the plate temperature differs greatly from that of the freestream and also from the results of [4], where always the leading edge has a freestream temperature. The calculated results in Fig. 3 have been obtained by assuming

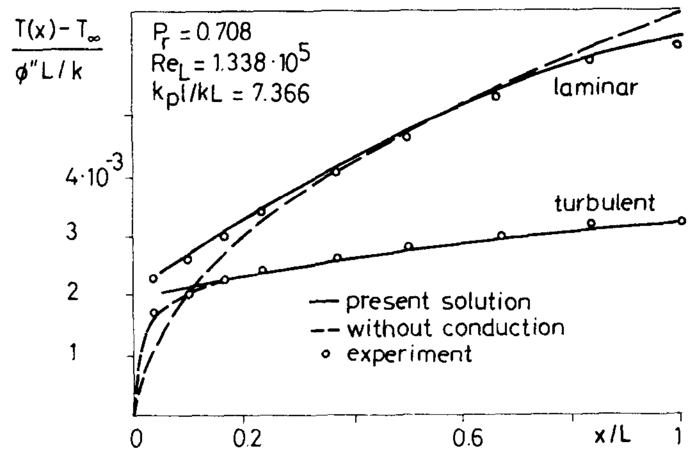

FIG. 3. Effect of conduction on the plate temperature.

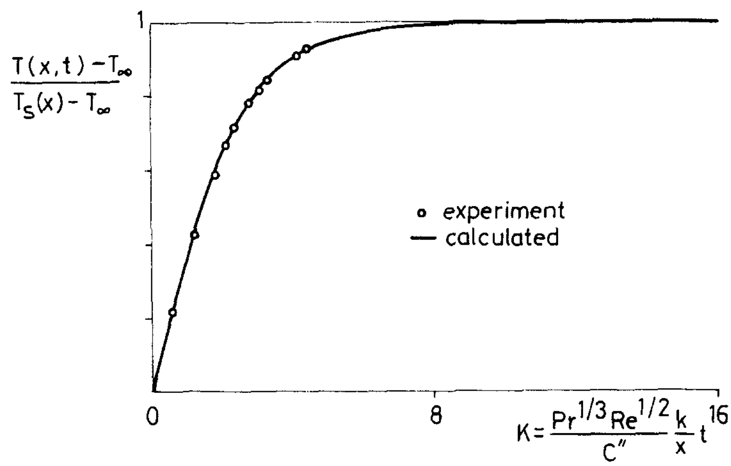

FIG. 4. Temperature response for a plate to a step change in heat flux. Laminar flow [9].

the edges of the plate insulated, and the first calculated point has been positioned at the beginning of the solid line.

4. TRANSIENT CASE

As an application of the previously presented method to transient case the temperature response of the plate subjected to a uniform step change in wall heat flux and cooled by convection is considered. Let us assume that the temperature distribution of the plate is known at a given instant (t). Then equation (5) gives the corresponding heat flux, and the heat-transfer coefficient can be calculated. For a small time increment the problem is treated as quasistatic assuming a constant heat-transfer coefficient. The temperature change ΔT_j at the point s_j during a small time increment Δt is obtained from

$$\Delta T_j = [T_j - T_\infty - \phi''/h_{cj}]\exp\left(-\frac{\Delta t h_{cj}}{C''}\right) + \phi''/h_{cj}. \qquad (12)$$

Using result (12) the temperature at the point s_j at instant $t + \Delta t$ is obtained and so on.

Figure 4 presents the plate temperature for a uniform step change in heat flux, when the initial temperature is the same as that of freestream and heat is transferred only by convection. We notice that the result for a laminar boundary layer can be presented by a single line. Also the experimental data for air, obtained by using an aluminium foil, 0.019 mm thick and stainless steel, 0.5 mm thick, are in agreement. Owing to the small values of emissivities in the plates and the used velocities, the corrections necessitated by radiation in experimental data were very small [9].

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STABILITY OF A LAYER OF FLUID SUBJECTED TO CONVECTIVE BOUNDARY CONDITIONS

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NOMENCLATURE

a, $= \lambda L$, dimensionless wave number;
g, acceleration of gravity;
h_1, h_2, heat-transfer coefficients;
H_j, $= h_j L/k$, Biot number, $j = 1, 2$;
k, thermal conductivity of fluid;
L, thickness of the fluid layer;
Pr, Prandtl number;
T, temperature;
$\bar{T}_{w1}, \bar{T}_{w2}$, mean temperatures of the lower and upper surface, respectively;
$T_{\infty 1}, T_{\infty 2}$, temperatures f the outside environments;
t, time;
w, velocity component in the z-direction;
z, coordinate normal to the walls.

Greek symbols
α, ν, thermal diffusivity and kinematic viscosity;
β^*, dimensionless rest-state temperature gradient;
η, $= z/L$, dimensionless coordinat ;
δ, angle measured from horizontal;
γ, coefficient of thermal expansion.

Superscripts
$'$, $\hat{}$, refer to disturbance quantities.

FOR AN infinite horizontal layer of fluid confined between two isothermal plates with lower plate hotter than the upper one, the transition takes place at a critical Rayleigh number 1708 [1–6]. There are numerous other factors that affect the initiation of convective flow patterns in the fluid [7–15]. In all of these and other investigations the boundaries confining the fluid are assumed at prescribed temperatures. Only in [16, 17] the problem of stability is considered for a horizontal layer only with one of the surfaces subjected to convective boundary condition and by neglecting the heat capacity of the walls. The purpose of this study is to investigate the stability of fluid confined between two inclined parallel layers subjected to general convective boundary conditions at both surfaces.

ANALYSIS

Consider a layer of fluid between two parallel plates subjected to a negative temperature gradient in the direction perpendicular to the plates (i.e. z-direction). Fluid is incompressible, Newtonian, the physical properties are constant except for the density which appears in the body force (i.e. Boussinesq approximation) and that the wall admittance is negligible. We restrict our analysis to the range of inclinations such that the stability in the conduction regime with longitudinal disturbances will be dominant (i.e. $\delta \simeq 75°$ for $Pe \simeq 0.7$) [6, 15].

The disturbance equations for the type of stability problem considered here are well known [15, 18]; they are written as

$$\left(\frac{\partial}{\partial t} - \nu\nabla^2\right)\nabla^2 w' = \gamma g \nabla_{xy}^2 T' \cos\delta \qquad (1a)$$

$$\left(\frac{\partial}{\partial t} - \alpha\nabla^2\right)T' = -\frac{d\bar{T}}{dz}w'. \qquad (2a)$$

Subject to the boundary conditions

$$w' = \frac{\partial w'}{\partial z} = 0 \text{ at } z = 0 \text{ and } z = L. \qquad (1b)$$

$$\begin{aligned} -k\frac{\partial T'}{\partial z} + h_1 T' &= 0 \text{ at } z = 0 \\ k\frac{\partial T'}{\partial z} + h_2 T' &= 0 \text{ at } z = L \end{aligned} \qquad (2b)$$

where for horizontal layer

$$\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}, \quad \nabla_{xy}^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

while $\partial/\partial x = 0$ for inclined layer;
and w' and T' are the disturbance quantities for velocity normal to the walls and the temperature $d\bar{T}/dz$ is the constant negative temperature gradient. The disturbance quantities w' and T' are written as

$$\begin{aligned} F'(x, y, z, t) &= F(z)\exp[i(k_1 x + k_2 y) + pt], \\ F &\equiv w \text{ or } T, \end{aligned} \qquad (3)$$

where

$$(k_1^2 + k_2^2)^{1/2} \equiv \lambda = \text{the wave number}$$

and p, in general, is a complex number, and $k_1 = 0$ for inclined layer.

Then the solution (3) is introduced into the system of equations (1) and (2); the resulting expressions in the dimensionless form are given as

$$[P - (D^2 - a^2)](D^2 - a^2)w^*(\eta) = -a^2\theta^*(\eta) \qquad (4a)$$

$$[PPr - (D^2 - a^2)]\theta^*(\eta) = Raw^*(\eta)\cos\delta \qquad (5a)$$